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Transmission Lines in Metamaterial

A Route to Design Artificial Metamaterials

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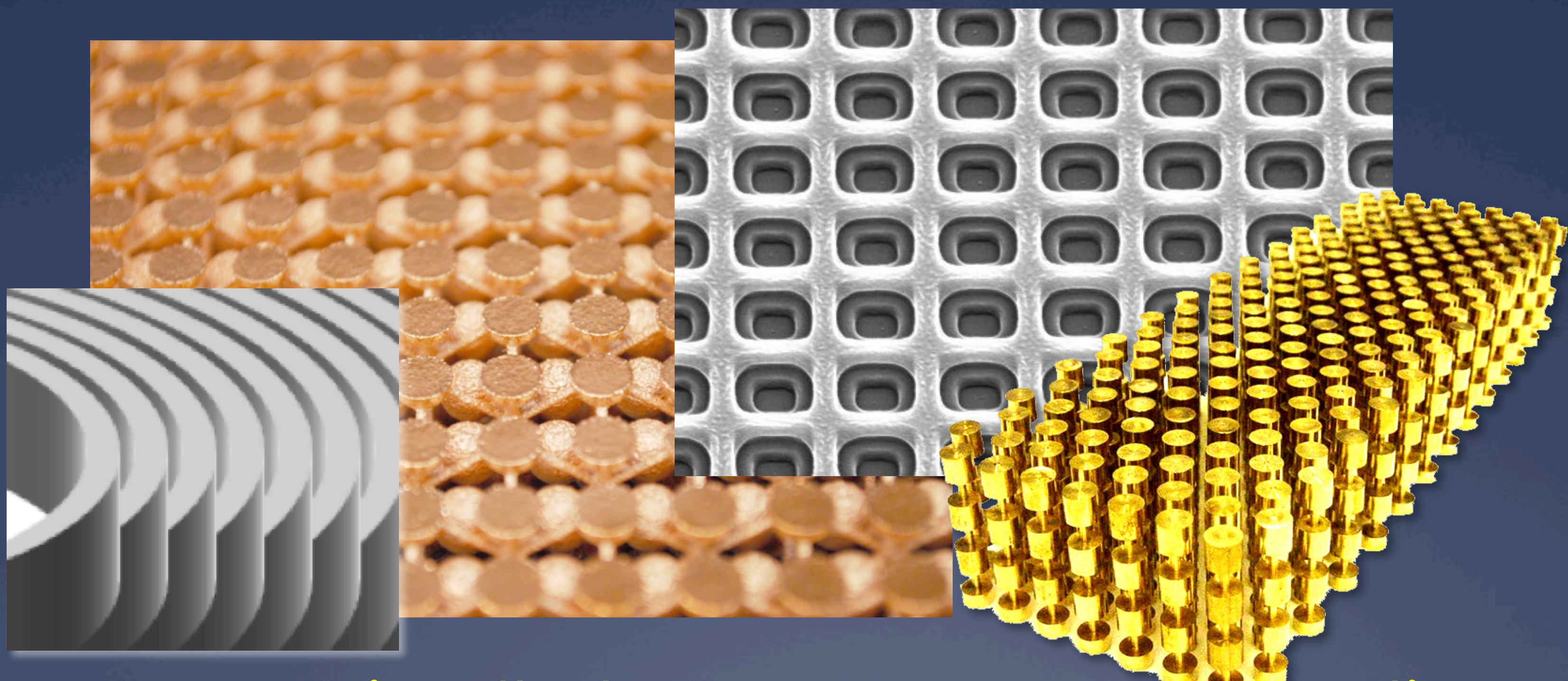
Outline

- Introduction - Artificial materials
- *Transmission line approach* for metamaterial design
- Design examples
- Conclusions

What Are Metamaterials?

- Artificial materials composed of sub-wavelength elements
- Exhibit unusual electromagnetic properties which do not exist in nature

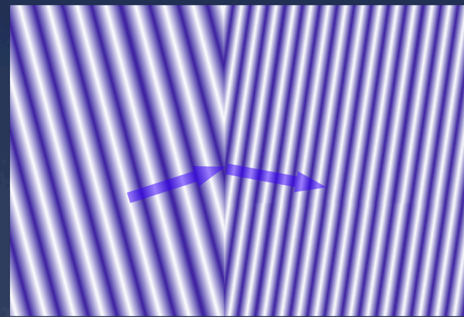


Macroscopically homogeneous media

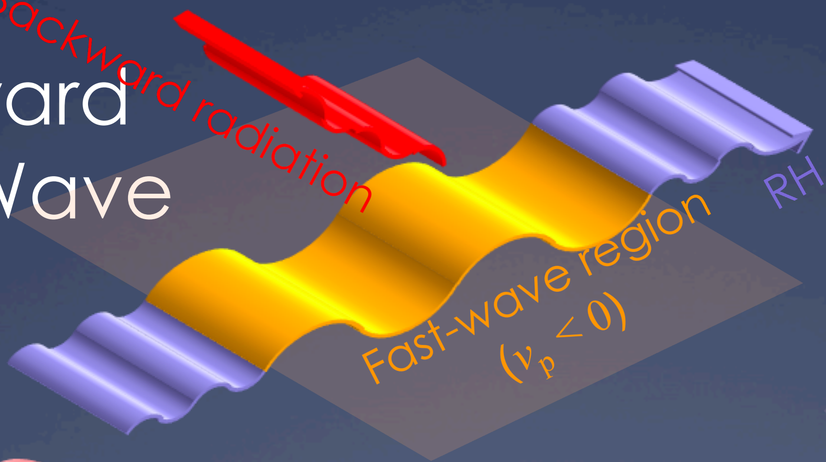
Unusual Properties

Invisible cloak

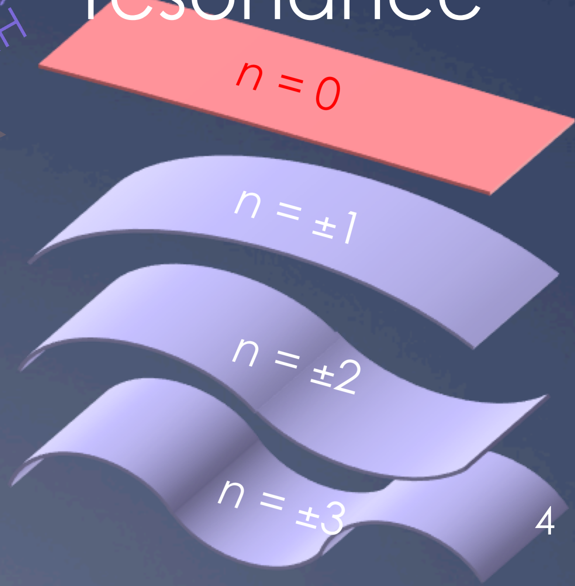
Negative refraction

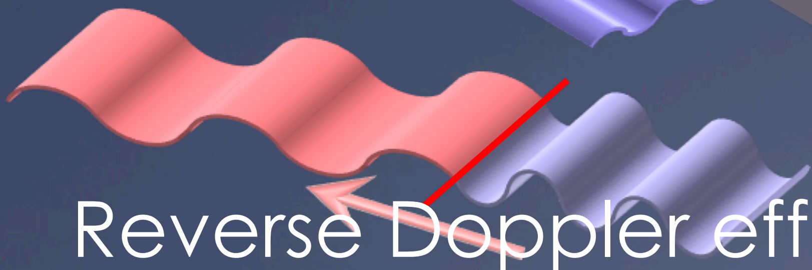


Backward Leaky-Wave



Zeroth-order resonance





Reverse Doppler effect

Designing Macroscopic Properties

- *Material*
  Material properties
- *Shape*
  Inductive/capacitive resonance
- *Arrangement*
  Couplings, periodicity

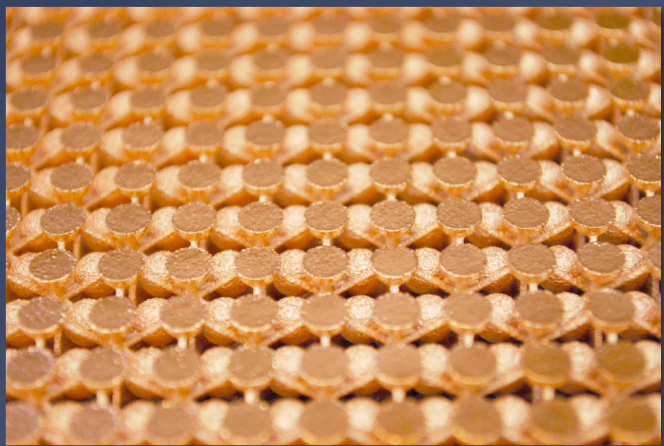
affect macroscopic material properties very much

Characterization

- ✓ Materials
- ✓ Topology
- ✓ Dimension
- ✓ Wavelength

→

- ✓ Negative/positive permittivity and permeability
- ✓ Dispersion
- ✓ Band structure
- ✓ Anisotropy
- ✓ Chirality



$$\boldsymbol{D} = \overline{\varepsilon}\boldsymbol{E} + \overline{\xi}\boldsymbol{H}$$

$$\boldsymbol{B} = \overline{\mu}\boldsymbol{H} + \overline{\zeta}\boldsymbol{E}$$

Can We Go Backward?

- ✓ Materials
- ✓ Topology
- ✓ Dimension
- ✓ Wavelength

?

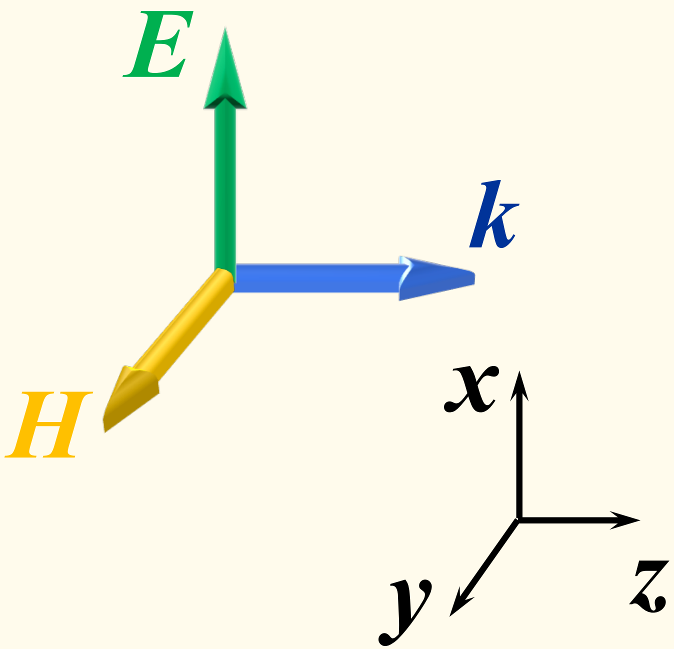
- ✓ Negative/positive permittivity and permeability
- ✓ Dispersion
- ✓ Band structure
- ✓ Anisotropy
- ✓ Chirality

$$\boldsymbol{D} = \bar{\varepsilon}\boldsymbol{E} + \bar{\xi}\boldsymbol{H}$$

$$\boldsymbol{B} = \bar{\mu}\boldsymbol{H} + \bar{\zeta}\boldsymbol{E}$$

Transmission Line Concept
A Route to Design Artificial Materials

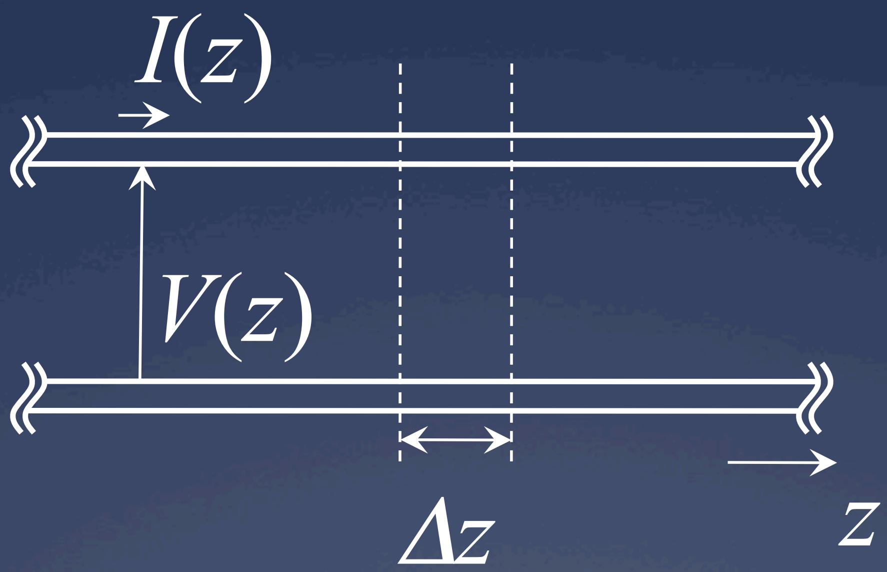
Plane Wave



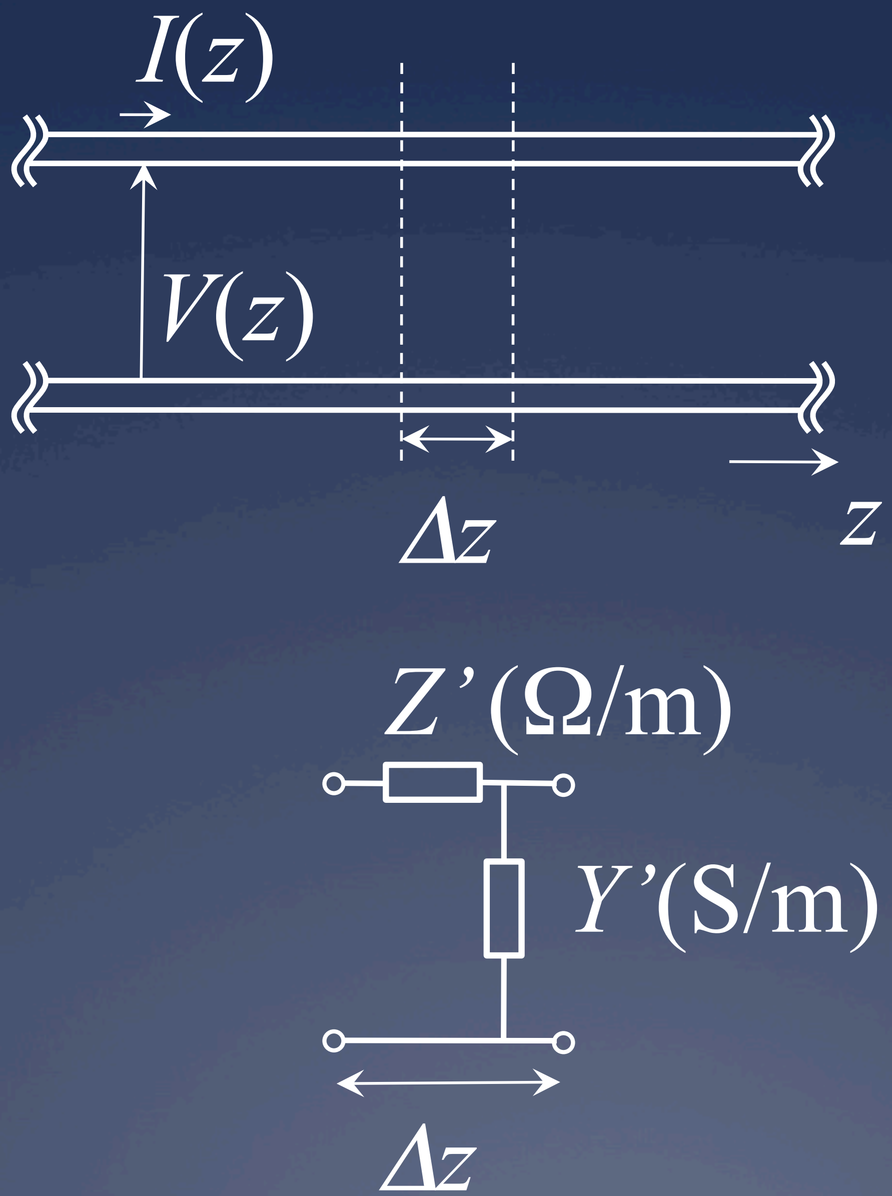
$$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y$$

$$\frac{\partial H_y}{\partial z} = -j\omega\varepsilon E_x$$

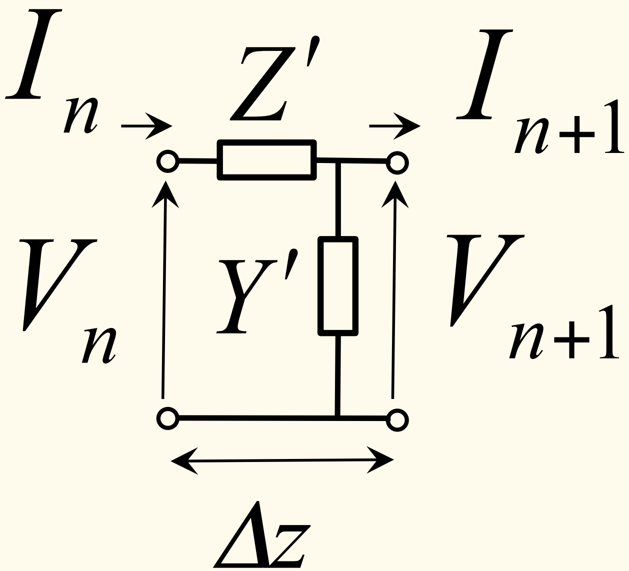
Transmission Line



Transmission Line



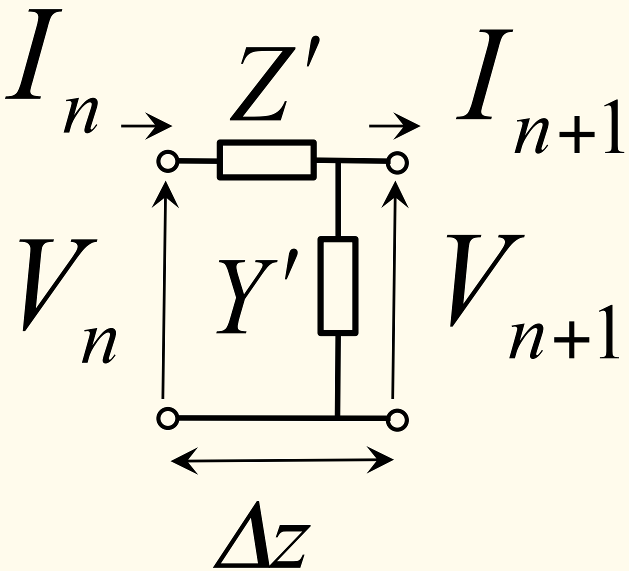
Kirchhoff's Laws



$$\frac{V_{n+1} - V_n}{\Delta z} = -Z' I_n$$

$$\frac{I_{n+1} - I_n}{\Delta z} = -Y' V_{n+1}$$

Telegrapher's Equations



$$\Delta z \rightarrow 0$$

$$\frac{\partial V}{\partial z} = -Z'I$$

$$\frac{\partial I}{\partial z} = -Y'V$$

(E,H) vs. (V,I) Correspondence

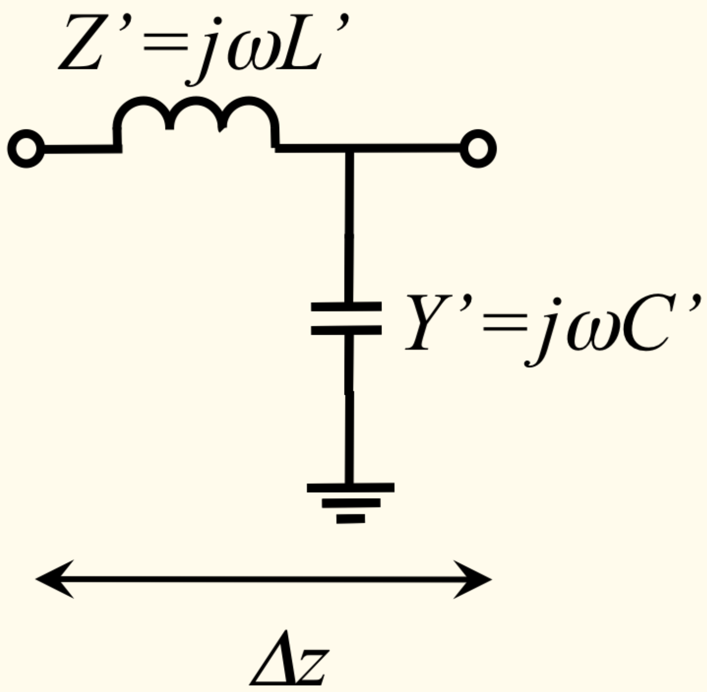
Maxwell's Eqs.	Telegrapher's Eqs.
$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y$	$\frac{\partial V}{\partial z} = -Z'I$
$\frac{\partial H_y}{\partial z} = -j\omega\varepsilon E_x$	$\frac{\partial I}{\partial z} = -Y'V$

(ε,μ) vs. (Z',Y') Correspondence

Maxwell's Eqs.	Telegrapher's Eqs.
$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y$	$\frac{\partial V}{\partial z} = -Z'I$
$\frac{\partial H_y}{\partial z} = -j\omega\varepsilon E_x$	$\frac{\partial I}{\partial z} = -Y'V$

Example: Right-Handed Material

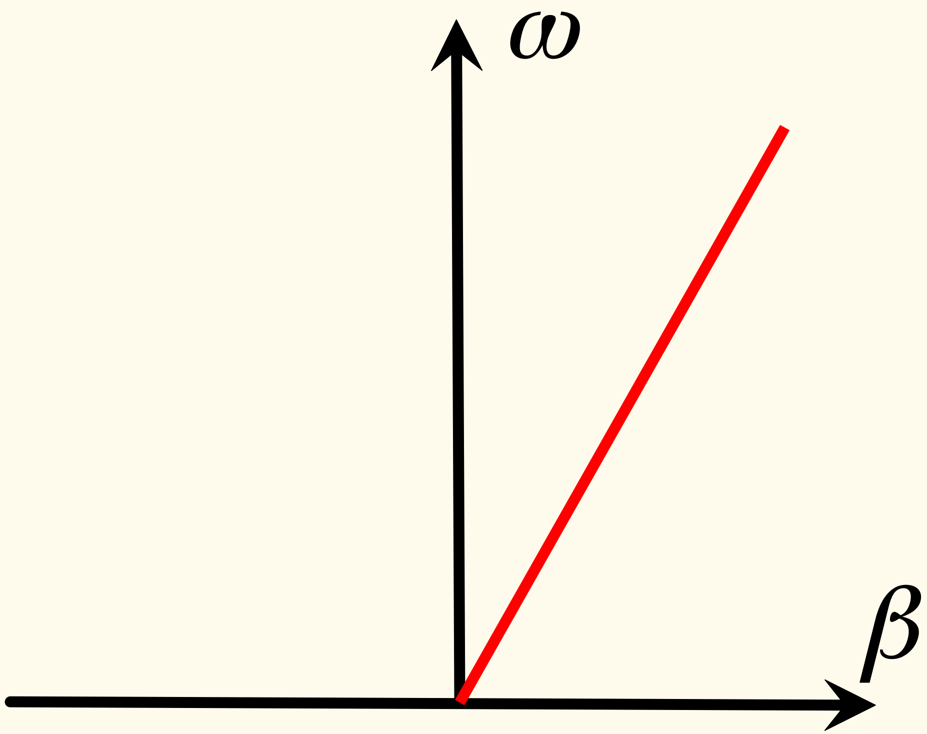
$$\varepsilon = \frac{Y'}{j\omega} \qquad \mu = \frac{Z'}{j\omega}$$



$$\varepsilon = C' \ \text{(F/m)}$$

$$\mu = L' \ \text{(H/m)}$$

Dispersion Relation and Impedance



$$\beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

Phase and Group Velocities

$$\beta = \omega\sqrt{L'C'}$$

$$v_{\mathrm{p}} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}$$

$$v_{\mathrm{g}} = \left(\frac{\partial\beta}{\partial\omega}\right)^{-1} = \frac{1}{\sqrt{L'C'}}$$

Plane Wave

$$\frac{d^2E_x}{dz^2} + k^2E_x = 0$$

$$\frac{d^2H_y}{dz^2} + k^2H_y = 0$$

$$j\omega\varepsilon,\ j\omega\mu$$

$$k = \omega\sqrt{\varepsilon\mu}$$

$$E_x = E_x^+e^{-jkz} + E_x^-e^{jkz}$$

$$H_y = H_y^+e^{-jkz} + H_y^-e^{jkz}$$

$$\eta = E_x^+/H_y^+ = -E_x^-/H_y^- = \sqrt{\mu/\varepsilon}$$

$$S_z = E_xH_y^*$$

Transmission Line

$$\frac{d^2V}{dz^2} - \gamma^2V = 0$$

$$\frac{d^2I}{dz^2} - \gamma^2I = 0$$

$$Y',Z'$$

$$\gamma = \sqrt{Z'Y'}$$

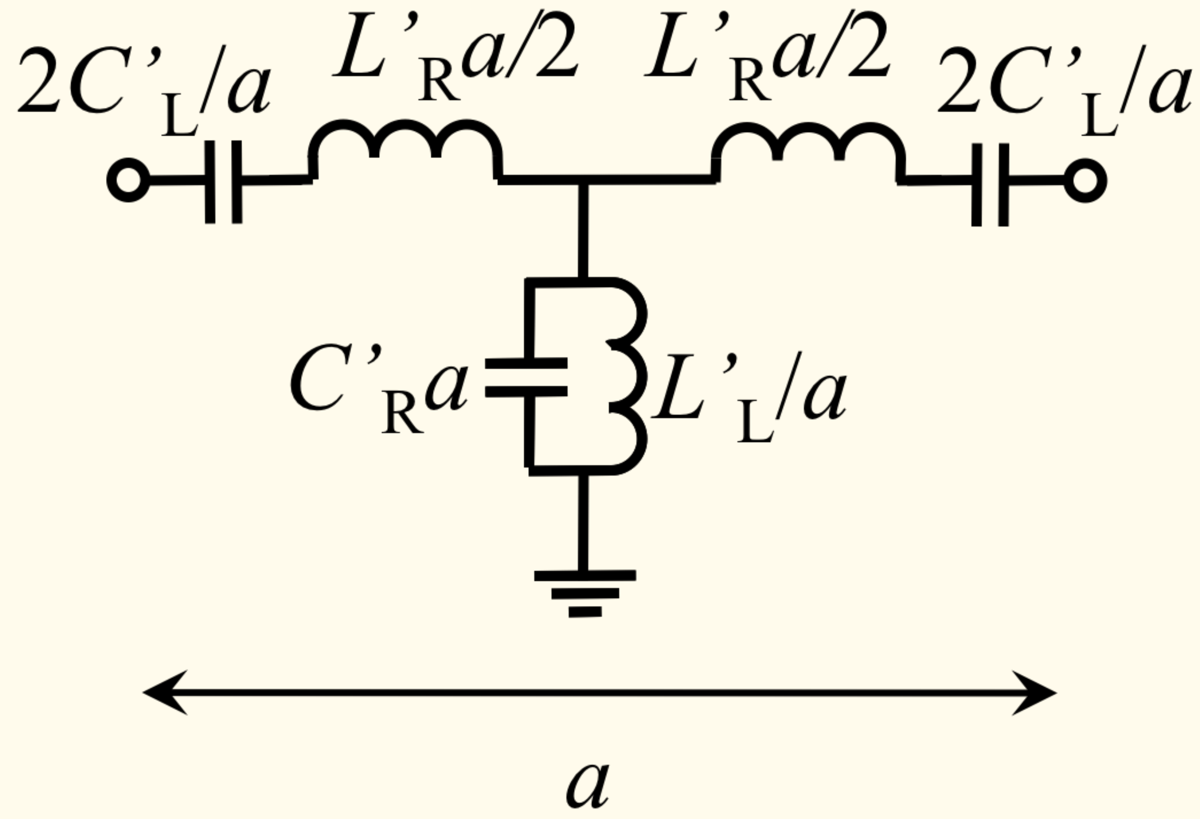
$$V = V^+e^{-\gamma z} + V^-e^{\gamma z}$$

$$I = I^+e^{-\gamma z} - I^-e^{\gamma z}$$

$$Z_0 = V^+/I^+ = -V^-/I^- = \sqrt{Z'/Y'}$$

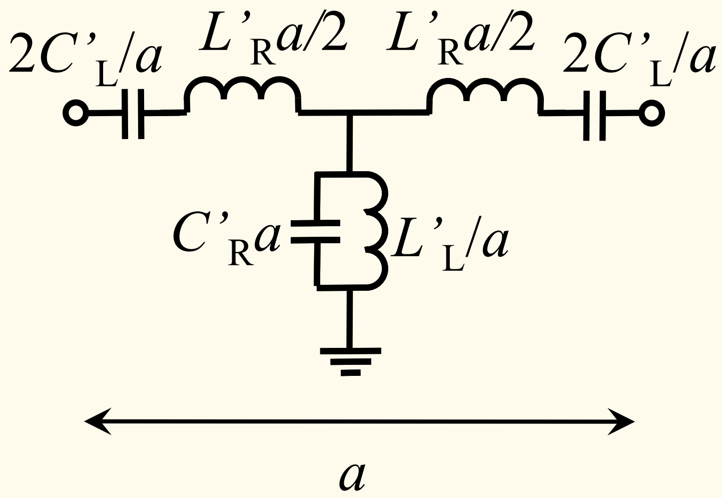
$$P = VI^*$$

Example 2: Left-Handed Materials



Periodic structure

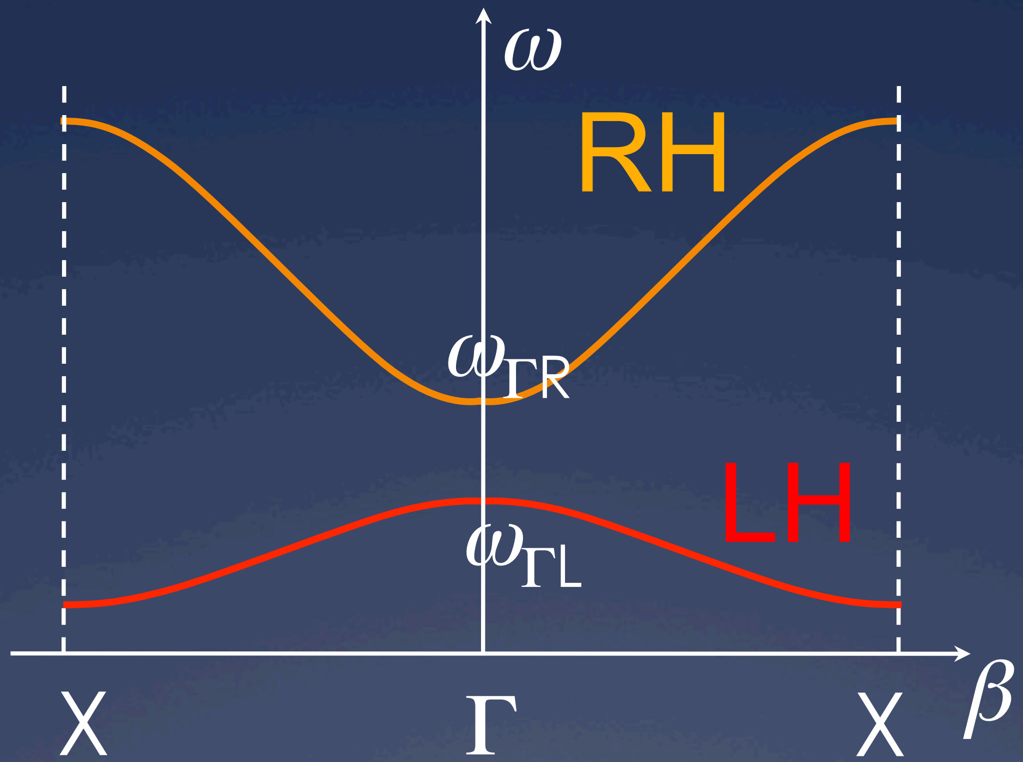
Dispersion Relation



$$\cos \beta a = 1 + Z(\omega)Y(\omega)$$

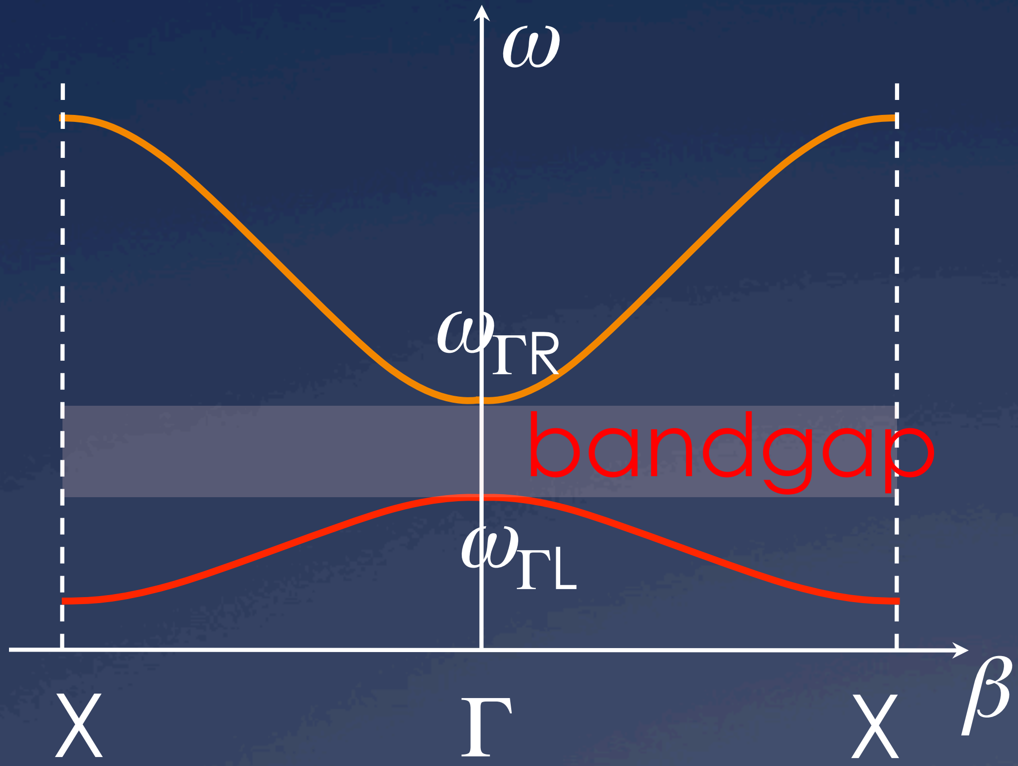
$$Z(\omega) = \frac{1}{2}\left(\frac{1}{j\omega C_{\mathrm{L}}} + j\omega L_{\mathrm{R}}\right) \quad Y(\omega) = \frac{1}{j\omega L_{\mathrm{L}}} + j\omega C_{\mathrm{R}}$$

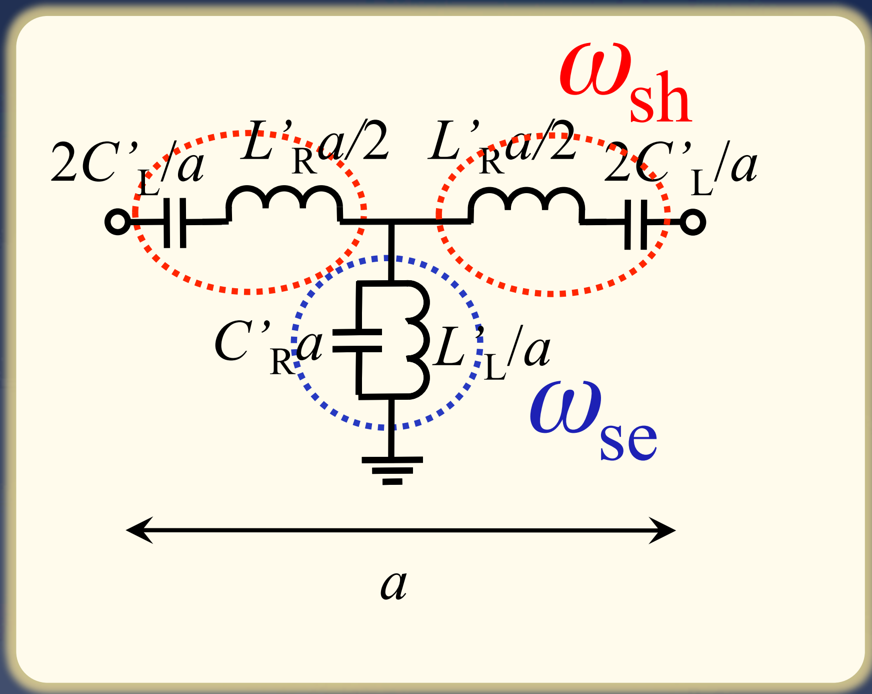
Dispersion Characteristics (Theory)



$$\cos \beta a = 1 + Z(\omega) Y(\omega)$$

Γ-Point Frequencies

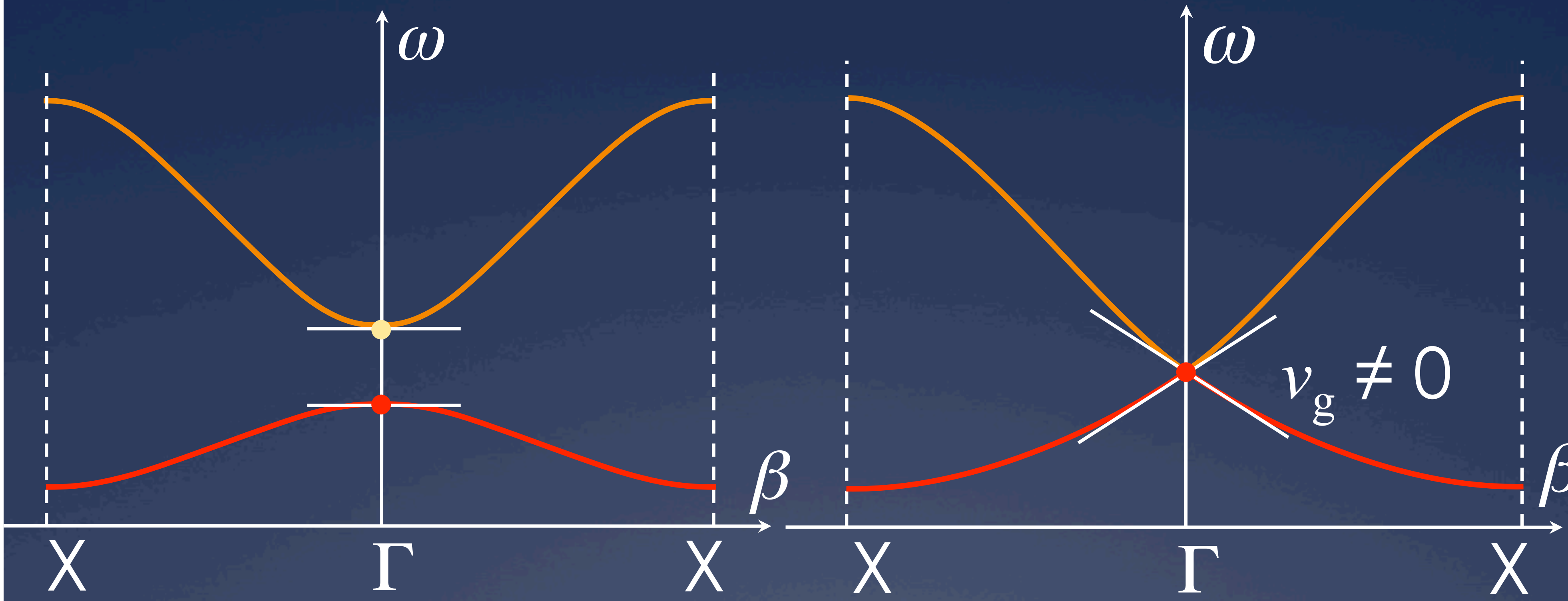




$$\omega_{\Gamma R} = \max(\omega_{se}, \omega_{sh})$$

$$\omega_{\Gamma L} = \min(\omega_{se}, \omega_{sh})$$

Closing the Bandgap

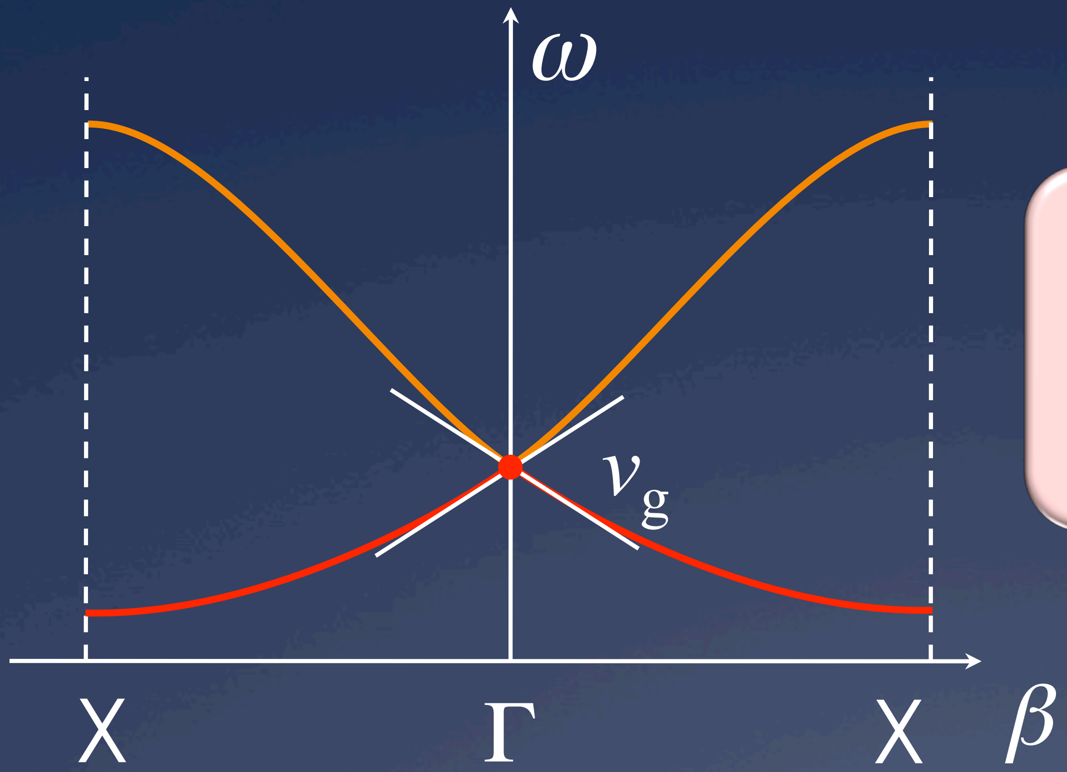


$\omega_{se} \neq \omega_{sh}$

$\omega_{se} = \omega_{sh}$

balanced condition

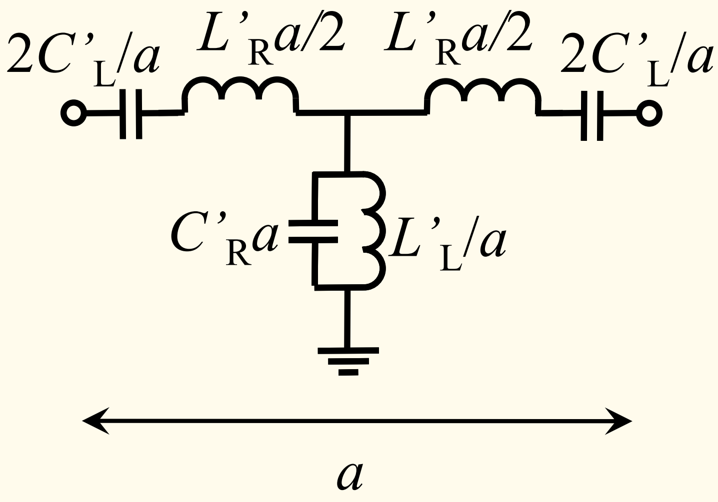
Non-Zero Group Velocity at Γ-Point



$$v_g\Big|_{\Gamma} = \pm \frac{1}{2\sqrt{L'_R C'_R}}$$

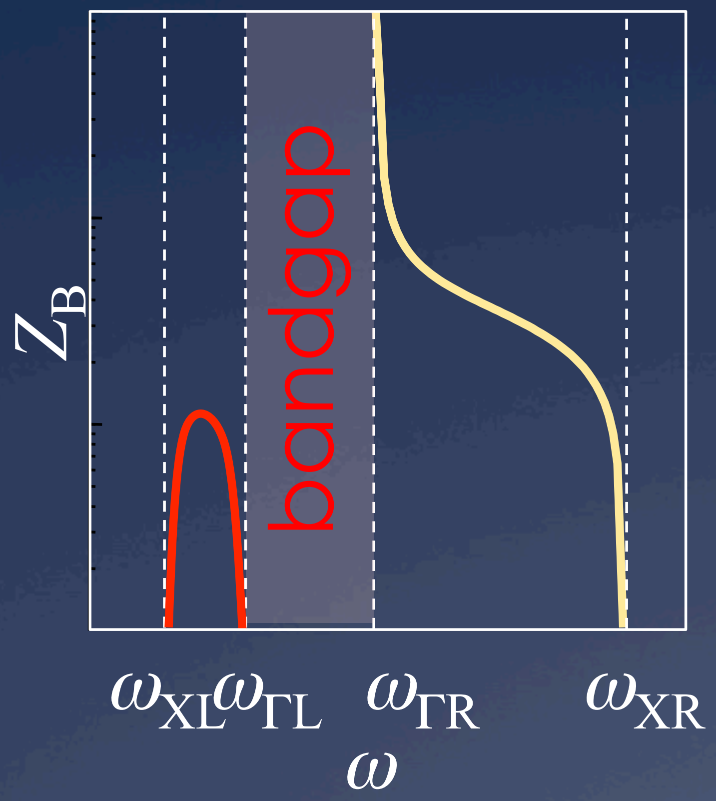
For $L'_R = \mu_0, \quad C'_R = \varepsilon_0,$ $\qquad v_g = \pm \dfrac{c_0}{2}$

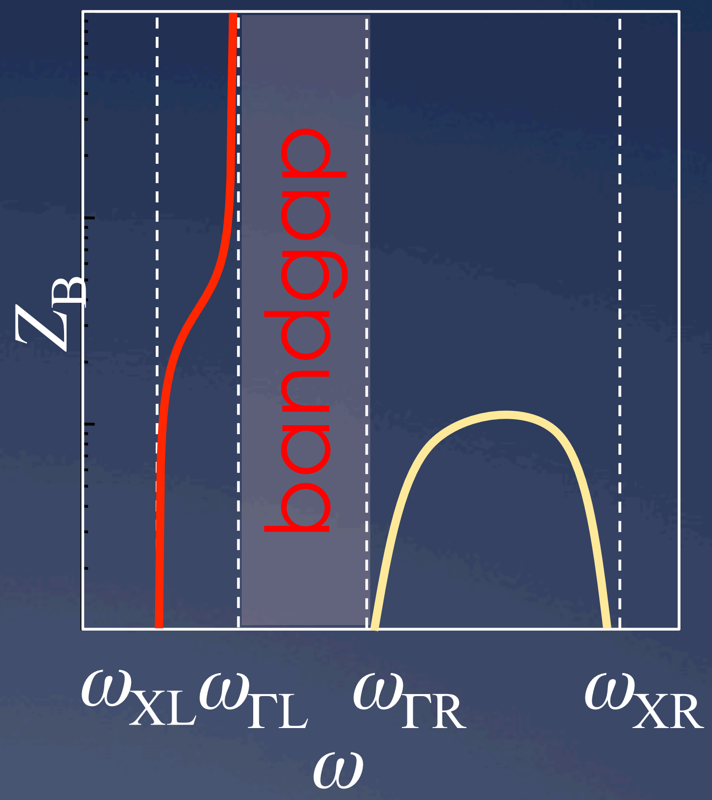
Bloch Impedance



$$Z_B = Z_L \sqrt{\frac{\dfrac{\omega^2}{\omega_{se}^2} - 1}{\dfrac{\omega^2}{\omega_{sh}^2} - 1} - \frac{\omega_L^2}{4\omega^2}\left(\frac{\omega^2}{\omega_{se}^2} - 1\right)^2} \qquad Z_L = \sqrt{\frac{L_L}{C_L}}$$

Frequency Dependence of Z_B

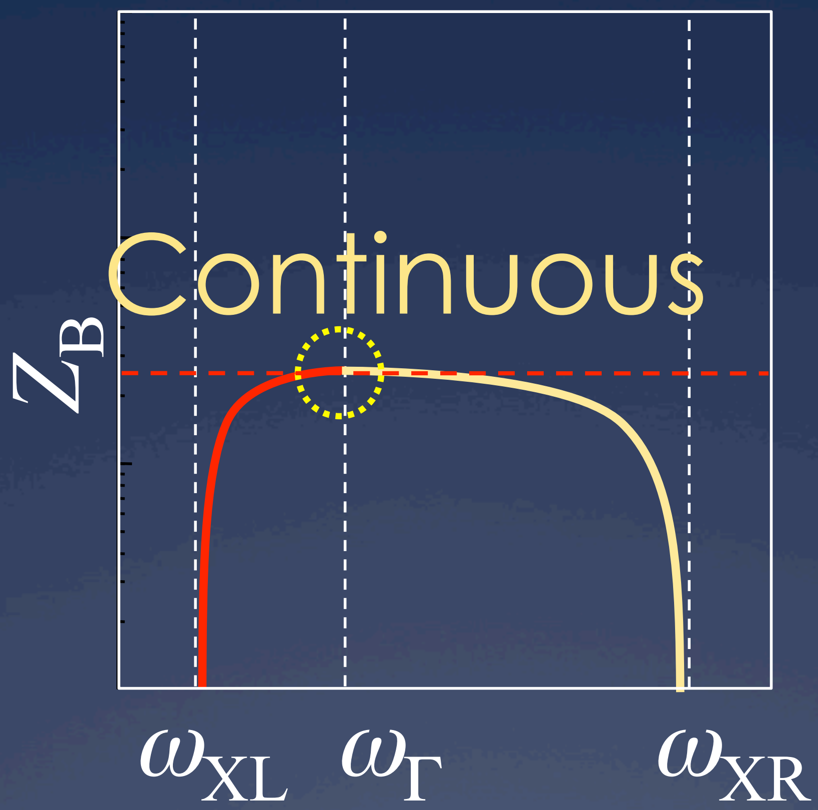




$\omega_{se} < \omega_{sh}$

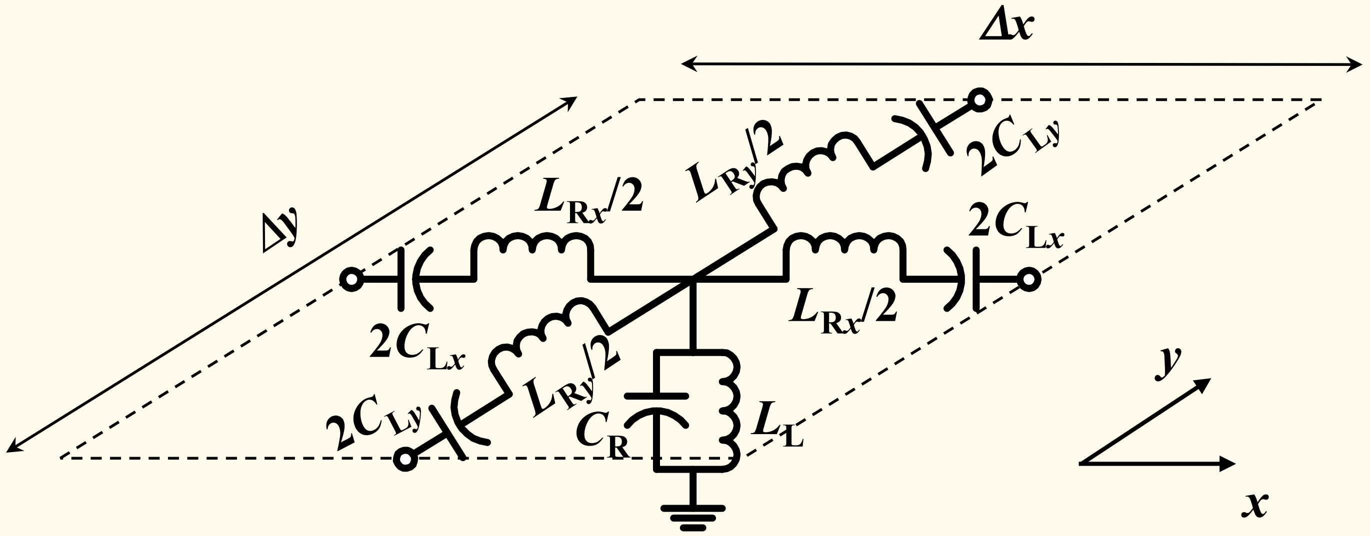
$\omega_{se} > \omega_{sh}$

Continuity of Z_B at the Γ-Point



$$\omega_{se} = \omega_{sh}$$

2D Extension

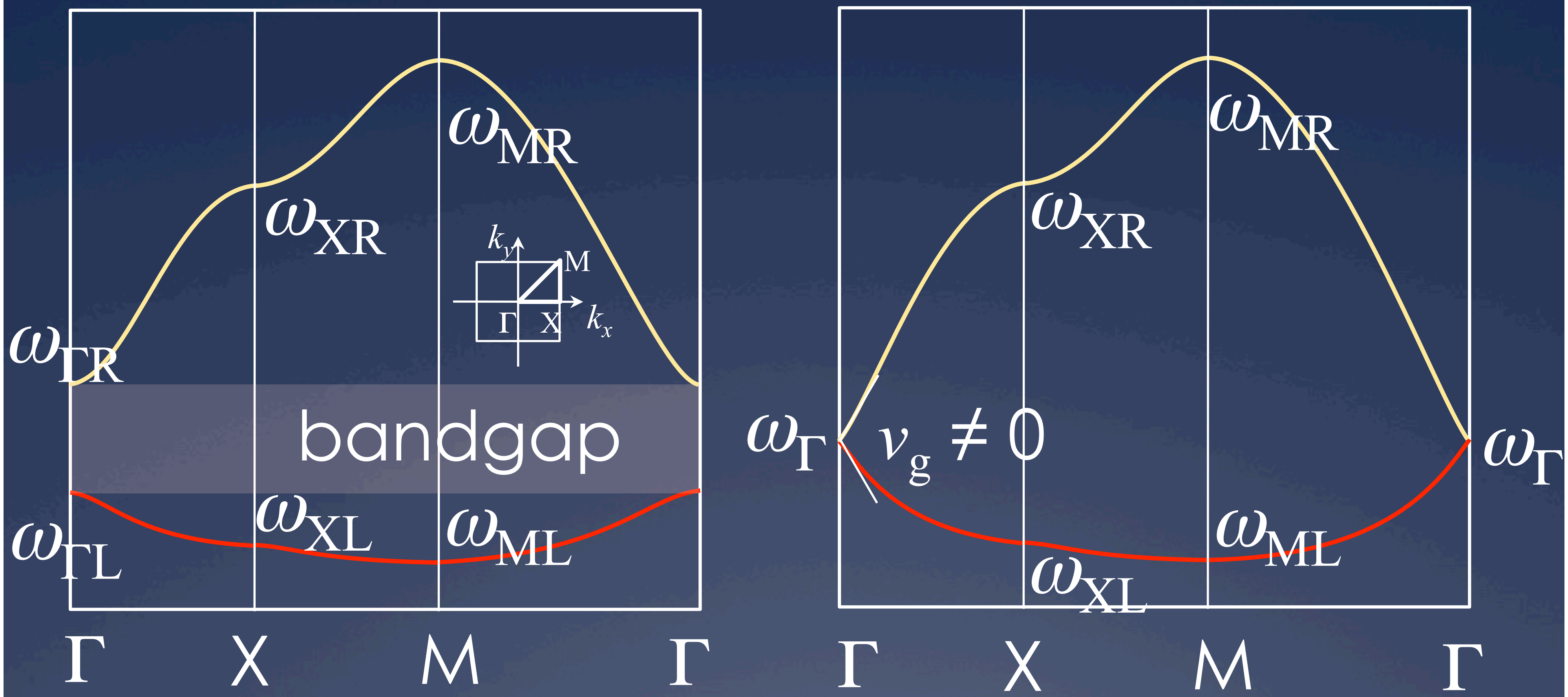


Unit cell

Dispersion Relation

$$\frac{\left(e^{-jk_x a}-1\right)^2}{e^{-jk_x a}}+\frac{\left(e^{-jk_y a}-1\right)^2}{e^{-jk_y a}}-2Z(\omega)Y(\omega)=0$$

Dispersion Characteristics (2D)



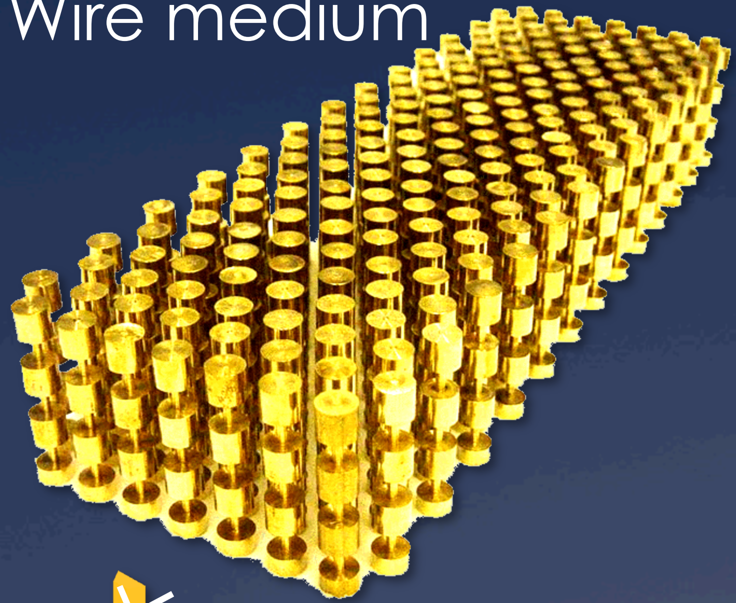
$\omega_{se} \neq \omega_{sh}$

$\omega_{se} = \omega_{sh}$

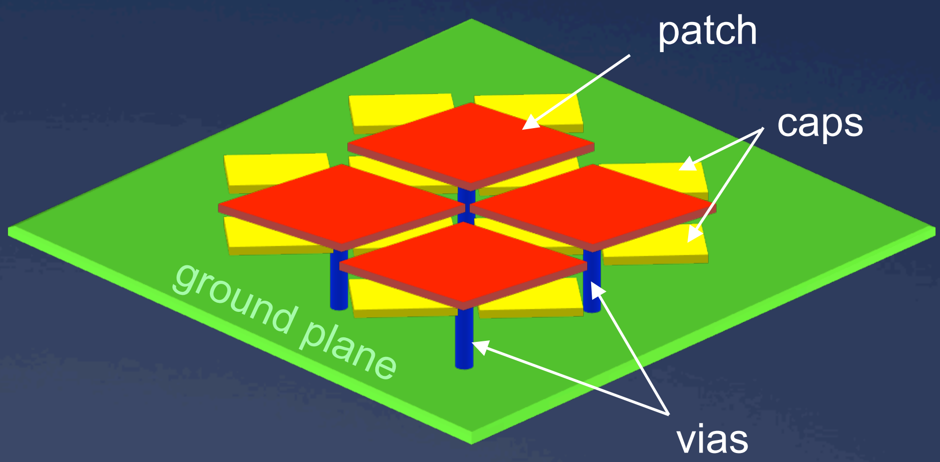
balanced condition

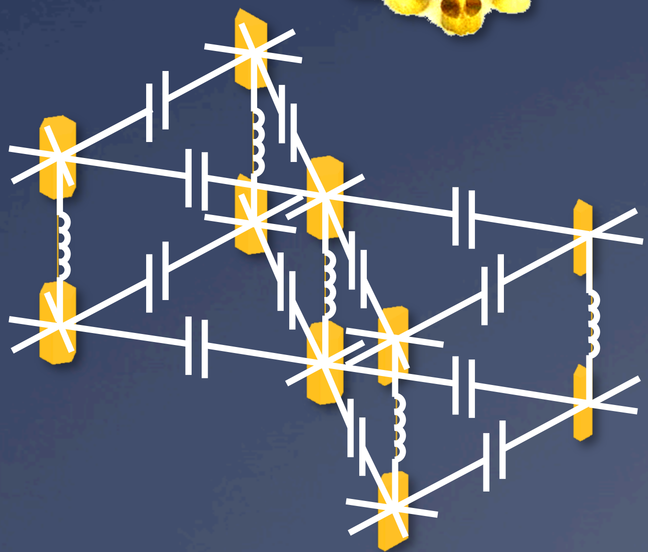
Implementation Examples

2D Wire medium

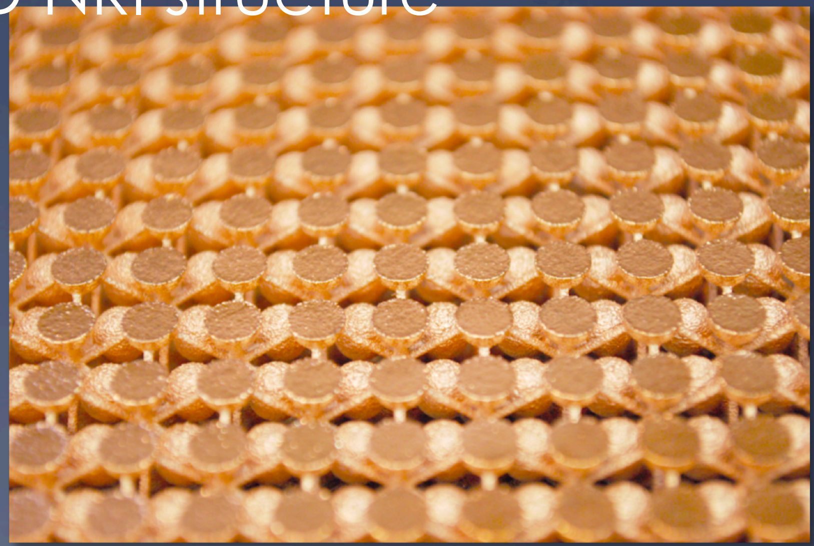


2D Mushroom structure





3D NRI structure

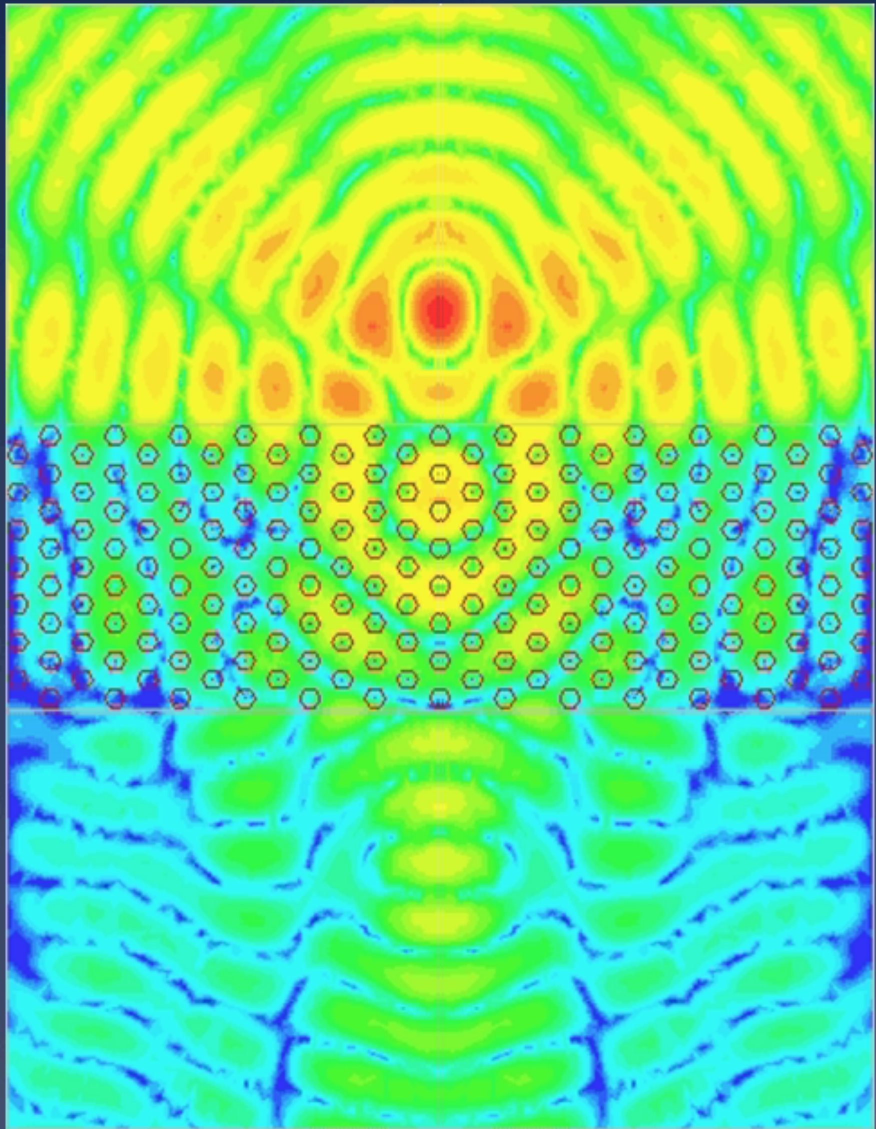


NRI Lens (Full-Wave Simulation)

RH

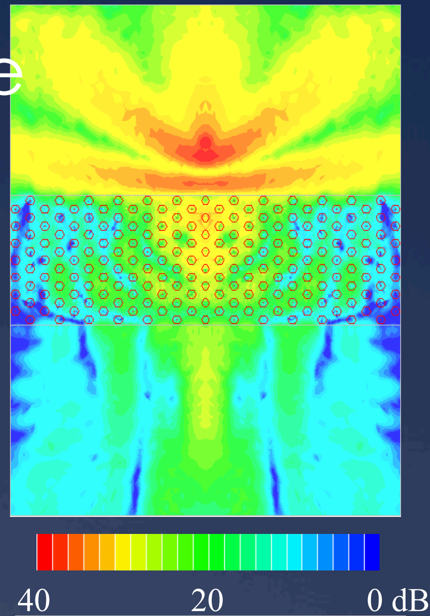
LH

RH



Electric field distribution

Amplitude



Phase

Conclusions

- The transmission line (TL) approach has been introduced as a route to design and engineer metamaterials
- The TL model can be an abstract form of artificial material revealing physical insight of wave propagation
- The TL model well describes properties of left-handed metamaterials including the band structure and impedance
- The concept can be extendable to 2D/3D structures
- The concept is scalable from microwave to optical frequency range