Non-reflectivity for circular polarization in chiral media and EIT phenomena in metamaterials

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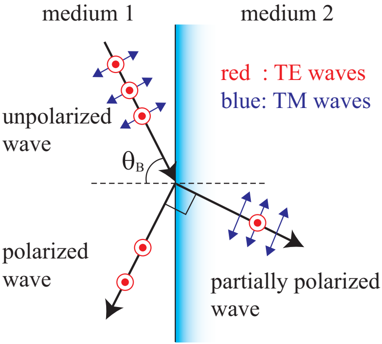
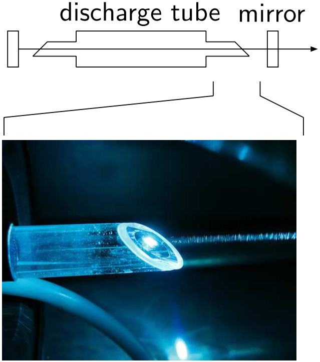
D.B. Brewster: Philos. Trans. Roy. Soc. Lond. **105** (1815).

- Suppression of reflection at a planar interface
  - — at a particular angle θ_B (Brewster's angle)
  - — only for TM mode (p waves)



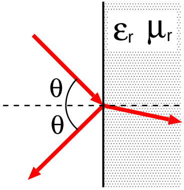


Brewster window of laser discharge tube

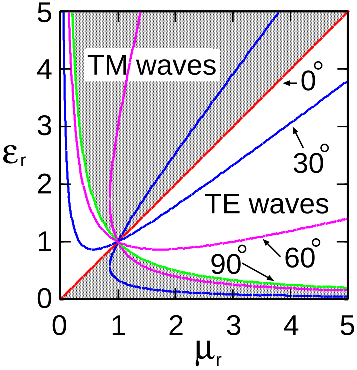
- $\underline{\varepsilon_r \neq 1,\ \mu_r = 1}$ (dielectric media)
  - TM (p) waves
  - Naturally occurring materials

  $\Updownarrow$ EM duality

- $\underline{\mu_r \neq 1,\ \varepsilon_r = 1}$ (magnetic media)
  - TE (s) waves
  - Metamaterials.



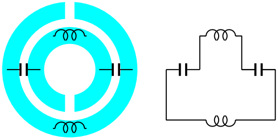
- Brewster's angle exists for (ε_r, μ_r) in
  - shaded area for TM (p) waves
  - unshaded area for TE (s) waves



Tamayama, Nakanishi, Sugiyama, and Kitano, Phys. Rev. B **73**, 193104 (2006)

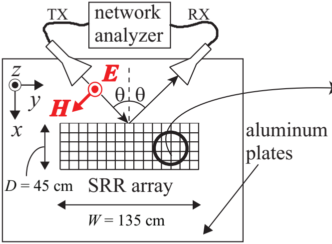
Split-ring resonator (SSR)

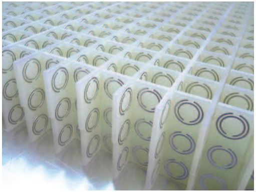
- Magnetic meta-atom ($\boldsymbol{B} \Rightarrow \boldsymbol{M}$)
  - Electromotive force: $\tilde{V} = \mathrm{i}\omega\tilde{B}S$ (S : loop area)
  - Loop current: $\tilde{I} = \tilde{V}[-\mathrm{i}\omega L - (\mathrm{i}\omega C) - -1 + R]^{-1}$
  - Magnetic moment: $\tilde{m} = \tilde{I}S$
  - Magnetization: $\tilde{M} = N\tilde{m}$ (N: number density)

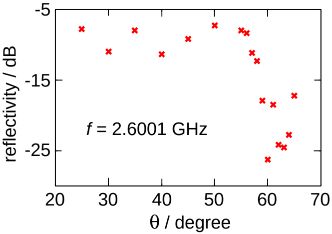


Pendry *et al.*: IEEE Trans. Microw. Theory Tech. **47**, 2075 (1999).

- 2D array of SRR (12672) in 2D (TE) waveguide.

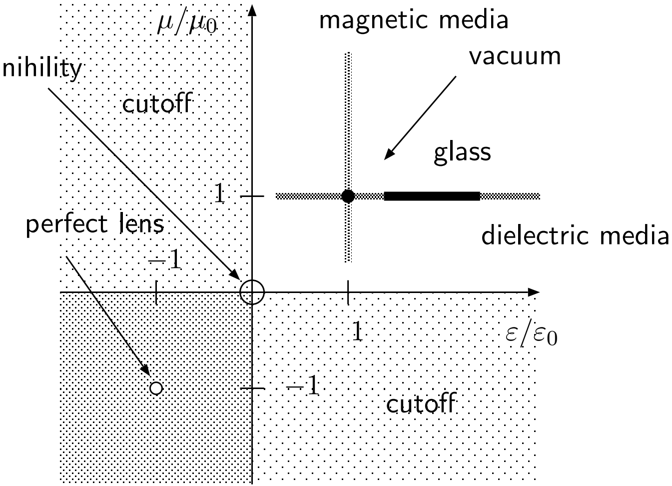






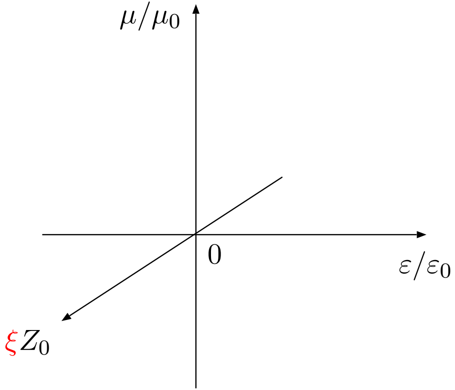
Power reflectivity is suppressed at a particular angle.

ϵ-μ plane (Veselago, 1968)

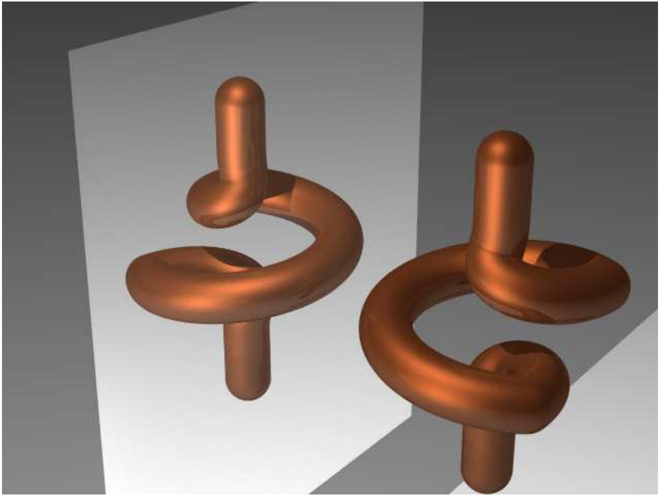


negative refraction $(n = \sqrt{\varepsilon_r}\sqrt{\mu_r} < 0)$

The 3rd dimension — chirality ξ



Chiral meta-atoms



$$\boldsymbol{E} \Rightarrow \boldsymbol{M}, \qquad \boldsymbol{B} \Rightarrow \boldsymbol{P}$$

No reflection condition for chiral media

- Chirality parameter: ξ

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} - \mathrm{i}\xi \boldsymbol{B}$$
$$\boldsymbol{H} = \mu^{-1} \boldsymbol{B} - \mathrm{i}\xi \boldsymbol{E}$$

- Wavenumber $k_\pm$ and wave impedance Z_c

$$k_\pm = \omega(\sqrt{\varepsilon\mu + \mu^2\xi^2} \pm \mu\xi), \quad Z_\mathrm{c} = \sqrt{\frac{\mu}{\varepsilon + \mu\xi^2}}$$

k_+: for LCP (left circularly polarized light)
k_-: for RCP (right circularly polarized light)

- No-reflection condition in terms of (ε, μ, ξ).

Tamayama, Nakanishi, Sugiyama, and Kitano: Opt. Express **16**, 20869 (2008)

Constitutive equations for chiral media

- Post representation (EB formalism)

$$\boldsymbol{D} = \varepsilon_{\mathsf{P}}\boldsymbol{E} - \mathrm{i}\xi_{\mathsf{P}}\boldsymbol{B}$$
$$\boldsymbol{H} = \mu_{\mathsf{P}}^{-1}\boldsymbol{B} - \mathrm{i}\xi_{\mathsf{P}}\boldsymbol{E}$$

- Tellegen representation (EH formalism)

$$\boldsymbol{D} = \varepsilon_{\mathsf{T}}\boldsymbol{E} - \mathrm{i}\mu_{\mathsf{T}}\xi_{\mathsf{T}}\boldsymbol{H}$$
$$\boldsymbol{B} = \mu_{\mathsf{T}}\boldsymbol{H} + \mathrm{i}\mu_{\mathsf{T}}\xi_{\mathsf{T}}\boldsymbol{E}$$

- Conversion

$$\varepsilon_{\mathsf{T}} = \varepsilon_{\mathsf{P}} + \mu_{\mathsf{P}}\xi_{\mathsf{P}}^2, \quad \mu_{\mathsf{T}} = \mu_{\mathsf{P}}, \quad \xi_{\mathsf{T}} = \xi_{\mathsf{P}},$$

- Magnetic constitutive relation
  - Sommerfeld (EB)

$$\boldsymbol{H} = \mu_0^{-1}\boldsymbol{B} - \boldsymbol{M}_{\mathsf{S}}, \quad \boldsymbol{M}_{\mathsf{S}} = \mu_0^{-1}\chi_{\mathsf{S}}\boldsymbol{B}$$

  - Kennelly (EH)

$$\boldsymbol{B} = \mu_0\boldsymbol{H} + \boldsymbol{M}_{\mathsf{K}}, \quad \boldsymbol{M}_{\mathsf{K}} = \mu_0\chi_{\mathsf{K}}\boldsymbol{H}$$

$\boldsymbol{M}_{\mathsf{S}}$ and $\boldsymbol{M}_{\mathsf{K}}$ are dimensionally different. In general, $\chi_{\mathsf{S}} \neq \chi_{\mathsf{K}}$.

- Specific permeability and susceptibility

$$\mu_{\mathsf{r}} = \frac{1}{1 - \chi_{\mathsf{S}}} = 1 + \chi_{\mathsf{K}}$$

EB vs EH

- The long confrontation between EB and EH formulations
  — It may be just a false dichotomy.
  - EB for media (the constitutive relations)

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$
$$\boldsymbol{H} = \mu_0^{-1} \boldsymbol{B} - \boldsymbol{M}$$

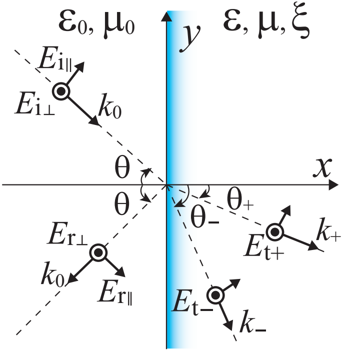
  - EH for waves (Maxwell's equations)

$$\mathrm{curl}\, \boldsymbol{H} = -\mathrm{i}\omega \boldsymbol{D}$$
$$\mathrm{curl}\, \boldsymbol{E} = \mathrm{i}\omega \boldsymbol{B}$$

Reflection Jones matrix

- Incident (i) and reflected (r) waves

$$\begin{bmatrix} E_{\mathrm{r}\perp} \\ E_{\mathrm{r}\parallel} \end{bmatrix} = \frac{1}{\Delta} M_{\mathrm{R}} \begin{bmatrix} E_{\mathrm{i}\perp} \\ E_{\mathrm{i}\parallel} \end{bmatrix}$$



- Reflection Jones matrix (2×2)

$$M_{\mathsf{R}} = c_{\mathsf{u}} I + c_2 \sigma_2 + c_3 \sigma_3$$
$$= c_{\mathsf{u}} I + c_\varphi \sigma_\varphi$$
$$\sigma_\varphi = \sigma_2 \sin\varphi + \sigma_3 \cos\varphi$$

- c_{u}, c_2, c_3, Δ are the functions of θ and (ε, μ, ξ).

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

σ_2, σ_3 :Pauli matrices

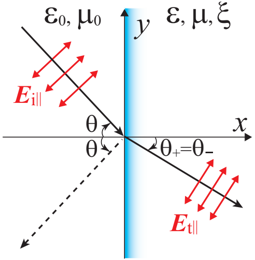
Eigenvalues and no reflection condition

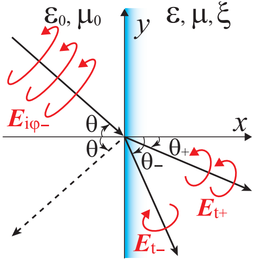
- A vanishing eigenvalue of M_{R} implies no-reflection.
  - The eigenvalue problem of M_{R} can be reduced to that of σ_φ.
  - An eigenvalue of M_{R} vanishes for $c_{\mathrm{u}} = c_\varphi$ $(c_{\mathrm{u}} = -c_\varphi)$.
- No-reflection is achieved by the corresponding eigen-polarization.

$$\boldsymbol{e}_{\varphi+} = \cos(\varphi/2)\,\boldsymbol{e}_z + \mathrm{i}\sin(\varphi/2)(\boldsymbol{e}_x \sin\theta + \boldsymbol{e}_y \cos\theta)$$
$$\boldsymbol{e}_{\varphi-} = \sin(\varphi/2)\,\boldsymbol{e}_z - \mathrm{i}\cos(\varphi/2)(\boldsymbol{e}_x \sin\theta + \boldsymbol{e}_y \cos\theta)$$

No reflection condition for achiral and chiral cases

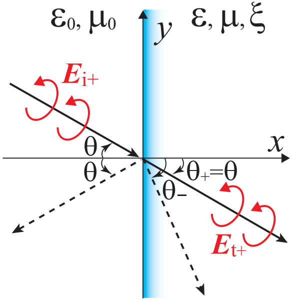
- Achiral ($\xi = 0$) case $\Rightarrow c_2 = 0$
  - $M_\mathsf{R} = c_\mathsf{u} I + c_3 \sigma_3$
  - Eigenpolarization: TM or TE
  - $c_\mathsf{u} = c_3$ ($c_\mathsf{u} = -c_3$) determines Brewster's angle.
- Chiral ($\xi \neq 0$) case
  - $M_\mathsf{R} = c_\mathsf{u} I + c_\varphi \sigma_\varphi$
  - Eigenpolarization: elliptical polarization (TE or TM-like)
  - $c_\mathsf{u} = c_\varphi$ ($c_\mathsf{u} = -c_\varphi$) determines Brewster's angle.





Matched impedance ($Z_c = Z_0$) case:

- $M_\mathsf{R} = c_\mathsf{u} I + c_2 \sigma_2$
- Eigenpolarization: Circular polarization (RCP or LCP)
- For $c_u = -c_2$ ($c_u = c_2$), the condition $\theta_+ = \theta$ ($\theta_- = \theta$), i.e., $k_+ = k_0$ ($k_- = k_0$) must be satisfied.
- Once, $k_\pm = k_0$ is satisfied, $c_u \equiv \mp c_2$ is met irrespective of the incident angle θ.
- $Z_c = Z_0$, $k_+ = k_0$ ($k_- = k_0$) $\Rightarrow$
  - No reflection and no refraction for left (or right) circular polarization
  - Independent of incident angle

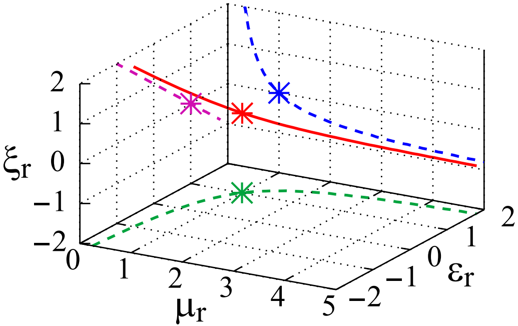


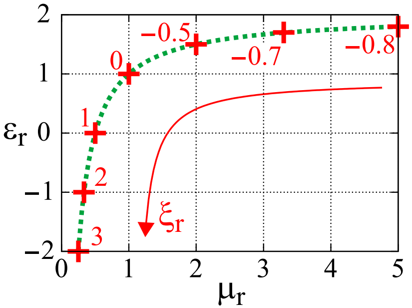
Medium parameters $\varepsilon,\ \mu,\ \xi$ for non-reflection

From $Z_c = Z_0,\ k_+ = k_0$,

For left circular polarization

$$\varepsilon_r = 2 - \frac{1}{\mu_r}, \quad \xi_r = -\left(1 - \frac{1}{\mu_r}\right) \quad (\varepsilon_r = \varepsilon/\varepsilon_0,\ \mu_r = \mu/\mu_0,\ \xi_r = Z_0\xi)$$

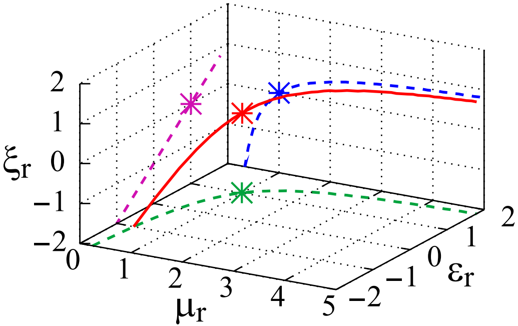


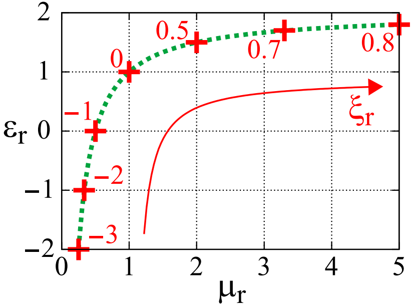


From $Z_c = Z_0,\ k_- = k_0$,

For right circular polarization

$$\varepsilon_r = 2 - \frac{1}{\mu_r}, \quad \xi_r = \left(1 - \frac{1}{\mu_r}\right) \quad (\varepsilon_r = \varepsilon/\varepsilon_0,\ \mu_r = \mu/\mu_0,\ \xi_r = Z_0\xi)$$





No reflection angles and polarizations

for a fixed chiral parameter: $\xi_{\mathrm{r}} = Z_0\xi = 0.2$

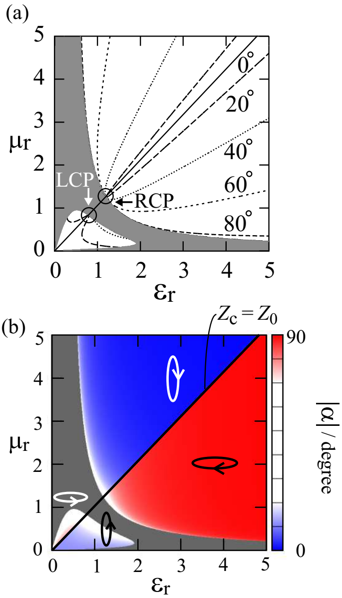
(a) No reflection angle θ_{B}

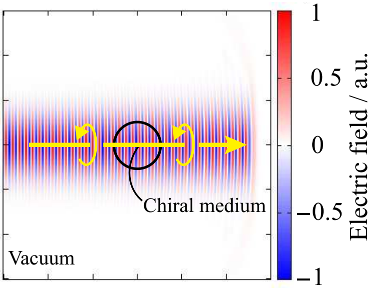
- Gray area: No no-reflection conditions exist
- Crossing points of contour lines $\rightarrow$ No-reflection for circular polarization

(b) No reflection (eigen-) polarization

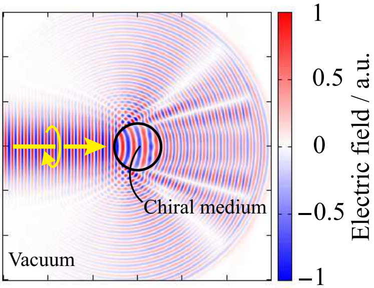
Ellipticity: $\alpha := \tan^{-1}(E_{\parallel}/\mathrm{i}E_{\perp})$

Red	TM-like
White	Circular
Blue	TE-like



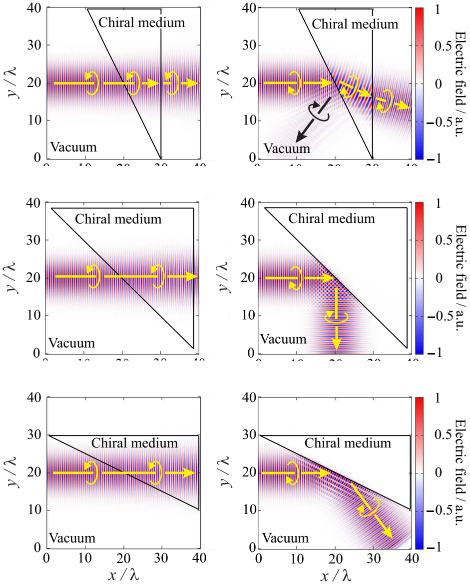

1
0.5
0
−0.5
−1
Electric field / a.u.
Chiral medium
Vacuum

LCP


1
0.5
0
−0.5
−1
Electric field / a.u.
Chiral medium
Vacuum

RCP

Circular Polarizing Beam Splitter



EIT-like mechanism

- Polarization $\boldsymbol{P}$ and magnetization $\boldsymbol{M}$

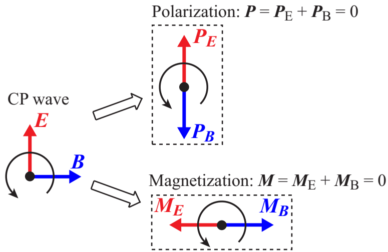
$$\boldsymbol{P} = \boldsymbol{P}_{\mathrm{E}} + \boldsymbol{P}_{\mathrm{B}}, \quad \boldsymbol{M} = \boldsymbol{M}_{\mathrm{B}} + \boldsymbol{M}_{\mathrm{E}}$$

Magnetically induced $\boldsymbol{P}_{\mathsf{B}}$, Electrically induced $\boldsymbol{M}_{\mathsf{E}}$.

- No-reflection condition for LCP

$$\varepsilon_{\mathrm{r}} - 1 = 1 - \mu_{\mathsf{r}}^{-1} = -\xi_{\mathrm{r}}$$

- Fields for LCP: $\boldsymbol{H} = \mathrm{i}Z_{\mathsf{c}}^{-1}\boldsymbol{E}$



- Combining these relations, we have

$$\boldsymbol{P} = 0, \quad \boldsymbol{M} = 0 \quad (\boldsymbol{P}_{\mathsf{E}} = -\boldsymbol{P}_{\mathsf{B}}, \quad \boldsymbol{M}_{\mathsf{B}} = -\boldsymbol{M}_{\mathsf{E}})$$

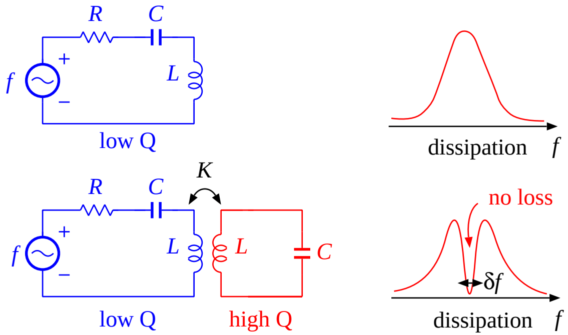
$\Longrightarrow$ equivalent to vacuum (only for LCP)

- Analogy to electromagnetically induced transparency (EIT)

Classical model of EIT

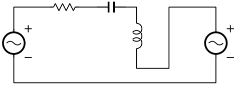
Electromagnetically induced transparency

- Coupled resonators   C. L. Garrido Alzar *et al.*:Am. J. Phys. **70**, 37, 2002



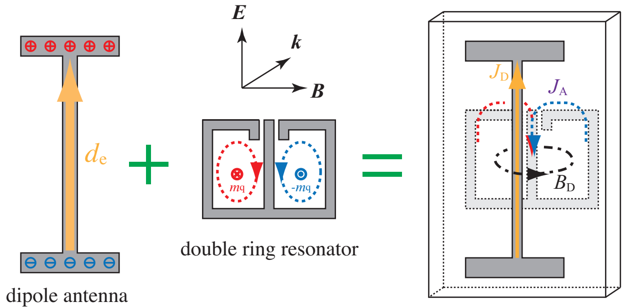
Coupling of low-Q and high-Q resonators: K

Width of transparent window: $\delta f \propto K$



An example of EIT meta atoms

Nakanishi *et al.*, Metamatarials 2009, London



- Low-Q resonator : Electric dipole

  — large radiation loss

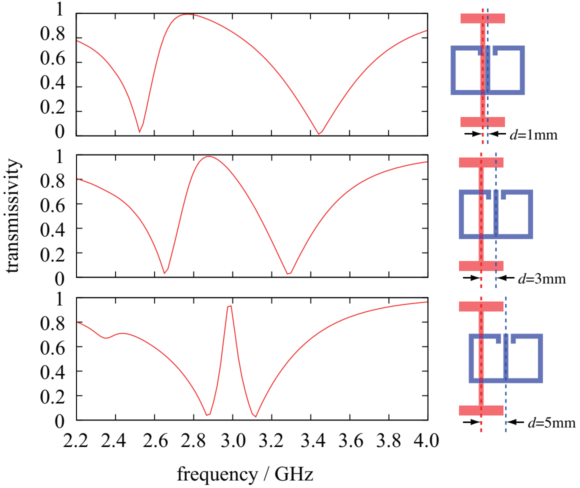
- High-Q resonator : Magnetic quadrupole

  — small radiation loss. coupling to low-Q resonator

The coupling can be controlled geometrically.

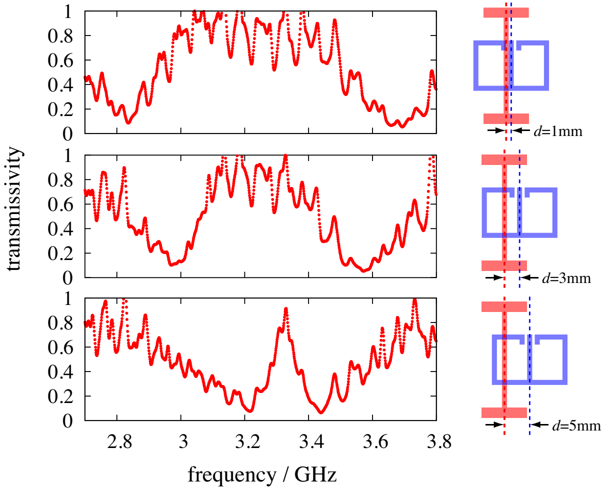
Simulation

- Transmittance



- EIT-like transmission — Narrower window for small coupling.

- Transmission



Conclusion

Summary

- Brewster no reflection effect for generalized media
  - Magnetic media (μ) — TE waves
  - Chiral media (ξ) — (TM or TE like) elliptical polarizations
- No reflection and no refraction for a circularly polarized light beam
  - Polarization-selective invisible medium

Future work

- Experiments in microwave region
- Implications in quantum optics — chiral vacuum
- Chirality due to spatially non-local response